Search Fatigue*

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March 3, 2018

ABSTRACT

Consumer search is not only costly but also tiring. We characterize the intertemporal effects that search fatigue has on monopoly and oligopoly prices, the product lines offered by firms, and the provision of consumer assistance (i.e., advice). These effects vary based on the type of search that occurs among stores and within them. In contrast to standard search models, accounting for fatigue leads to product proliferation and time-varying prices, cyclical price dispersion, and consumer assistance. We analyze the welfare implications of search fatigue and discuss under which conditions product proliferation and cyclical price dispersion are most pronounced.

*We thank Ricardo Alonso, Antonio Bernardo, Simon Board, Vince Crawford, Phil Leslie, Steven Lippman, Konstantin Milbradt, Martin Oehmke, Heikki Rantakari, Richard Saouma, Joel Sobel, Ivo Welch, Jidong Zhou, and seminar participants at UCLA, Yale, the Southwest Economic Theory conference, the Econometric Society North American Summer Meetings, and the USC-UCLA Finance Day for their thoughtful suggestions.

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1 Introduction

It is well-accepted in economics that search is costly unless you are a "shopper" with zero search costs (e.g., Varian, 1980; Stahl, 1989). But consumers may find it difficult to become a shopper if firms engage in obfuscation (e.g., Spiegler, 2006), shroud attributes (Gabaix and Laibson, 2006), create price complexity (Carlin, 2009), strategically release information and restrict advertising (Campbell, Mayzlin, and Shin, 2017), manipulate the layout of product offerings (Gu and Liu, 2013), or excessively proliferate products (Hämäläinen, 2017).[1]

However, the fact that search is also tiring (e.g., Spears, 2010) adds a further intertemporal consideration to a consumer's ability to become a shopper.[2] Search in one period may induce fatigue and inertia in subsequent periods, especially when the task of identifying the optimal choice is challenging. When firms intentionally make search more difficult, this not only affects a consumer's current purchase decision, but also her willingness to participate in active search later on. As a result a consumer may choose to become a shopper in some periods, but not in others.

In this paper, we study oligopoly behavior when fatigue from searching in one period may affect consumers' costs and incentives to become informed in future periods. Search occurs between stores and within them as in Hämäläinen (2017). We endow the firms with the ability to engage in wasteful artificial product differentiation (e.g., Grubb, 2015), whereby they may produce superfluous goods to make the consumer's search more arduous.[3] Indeed, large product lines and multiple offers appear to be effective in tiring consumers during search and causing inertia and suboptimal choices (Fasolo, Hertwig, Huber, and Ludwig, 2009; Kuester and Buys, 2009).

We show that, whereas a monopolist would always choose to produce only one valuable product, firms competing in an oligopoly engage in socially wasteful product proliferation. But the amount of product proliferation that occurs depends on the search technology. The more discriminating a

[1] See also Wilson (2010), Carlin and Manso (2011), Ellison and Wolitzky (2012), and Petrikaite (2014).

[2] The popular press has termed thisi "daily deal fatigue" (Hurban, 2012) and "search engine fatigue" (Sterling, 2007), while academics have called it "feature fatigue" (Thompson, Hamilton, and Rust, 2005) and "shopping fatigue" (Mitchell and Papavassiliou, 1999).

[3] As will become clear, our findings can generalize to any channel through which the firms may engage in obfuscation. Focusing on excessive product proliferation yields concrete empirical predictions. However, our analysis also applies to other related phenomena such as a lack of clear disclosure, price complexity, or any other mechanism through which firms may preserve rents in the face of competition.

consumer is in her search, the more artificial differentiation is necessary to wear her out during her search process. Sequential search leads to more product proliferation than all-or-nothing search. In fact, when the consumer sorts products one-by-one (i.e., sequential search across and within stores), the firms respond by engaging in a "finding the needle in a haystack" game in which product proliferation is the most severe.

In response to the added effort required during search, it might seem intuitive that a consumer would pace herself and smooth her search over time when facing a stream of lifetime choices. However, this does not arise in equilibrium in our model. Instead, it is optimal for the consumer to engage in time-varying search, whereby she exerts effort intermittently and rests following periods of active search.

When the consumer is not fatigued, she searches in the usual sense, prices are competitive, and no consumer assistance is offered. During these periods, the consumer earns all of the surplus. However, while she recovers, she chooses products randomly, prices are at monopoly levels, and the firms assist the consumer if she visits their store. Cycles therefore arise with time-varying prices and rent extraction. As we discuss in the paper, these cycles are not only robust to the type of search technology, but also to the specific form of how fatigue affects future costs. This is because the firms always have an incentive to produce enough products to ensure that the consumer becomes sufficiently fatigued when searching and she has to give up future surplus while she recovers.

The results in this paper apply to markets in which consumers search intermittently and repeatedly for the best alternative: money management, travel, shipping services, loans and credit terms, health insurance, and retail electricity. For example, Ericson (2014) shows that product proliferation, time-varying prices, and cycles of consumer inertia are consistent with the market for Medicare Part D insurance. Likewise, Hortaçsu, Madanizadeh and Puller (2017) document significant time-varying search costs and intermittent inertia in the Texas retail electricity market. Product proliferation has been cited as a root cause in this and other energy markets and regulation has been proposed to limit what is offered.[4]

[4] In 2011, the UK energy regulator Ofgem found that there were "too many confusing tariffs, making it harder for consumers to shop around for the best deal" and proposed to restrict each energy provider to offer only one tariff per payment method. Likewise, consumer advocates such as powertochoose.org have warned consumers that many companies use low promotional rates to pull customers into variable-rate plans and then hike prices over time.

To ease the reader's search for our salient ideas and to prevent fatigue, we provide the following outline for the paper. In Section 2, we describe the setup of our base model of search fatigue and analyze the oligopoly behavior in several search settings. Section 3 reconsiders our analysis with a continuum of consumers and heterogeneous fatigue. We also analyze an extension with brand loyalty. Section 4 concludes.

2 Product Proliferation and Fatigue

2.1 One-Consumer Model

Consider a market in which each firm (store) offers a line of products to a representative consumer over an infinite horizon. Time evolves in discrete periods indexed by $t \in \{0, \ldots, \infty\}$, and firm profits and consumer surplus are discounted by δ per period.

2.1.1 Firms

Each firm is indexed by $j \in N = \{1, ..., n\}$ and makes three strategic choices throughout the game: the length ℓ_j of its product line, prices for all of its products, and how much assistance to offer.

Each firm chooses ℓ_j once at $t = 0$ and pays a fixed cost $\kappa > 0$ per product in its line, which can be considered an upfront design or marketing cost. Every product line must contain a single product of fixed value $\bar{q}$. We refer to this as the *special* product. However, every firm also has discretion to offer additional products of zero value. Without loss of generality, we set the per-period marginal production cost for all goods to zero. Also, we assume that κ is arbitrarily small. This allows us to pin down equilibrium quantities and abstract away from the coordination problems that arise in joint production problems.[5]

In each period $t \geq 1$, every firm j chooses prices for each of its products p_j^m, where $m \in \{1, ..., \ell_j\}$. Let p_j^1 denote the price of firm j's special product. Additionally, in each period $t \geq 1$, each firm may offer assistance $a_j \in \{0, 1\}$ at no cost if the consumer visits its store. When $a_j = 1$, the consumer is directed to the special product. When a firm chooses $a_j = 0$, the consumer is left to

[5] If we set $\kappa = 0$, the firms will be indifferent between producing any number of goods greater or equal to the equilibrium quantities we derive. The necessary upper bound on κ is different based on the specifications in each version of our model and search technology.

make choices on her own.

2.1.2 Consumer Search

We allow search among stores to be either all-or-nothing (e.g., Varian, 1980) or sequential in nature (e.g., Stahl, 1989). In the all-or-nothing setting, if the consumer elects to search, she becomes informed about all of the firms. With sequential search, the consumer can search through stores one-by-one.

In each period $t \geq 1$, the consumer can pay a fixed cost to become a *shopper* in which her search costs are zero or she can remain a *non-shopper*, stay uninformed and make choices randomly. In classic search models these two types are given exogenously, but here they arise endogenously. The consumer may switch types over time based on market conditions and her energy reserves.

To capture the idea that search is tiring and has an intertemporal effect, we assume that the cost of becoming a shopper in any period t is $c(x_{t-1})$, where x_{t-1} is the number of products the consumer examined in the prior period. Therefore, search today affects the consumer's choices in the future and possibly firm behavior. We assume that $x_0 = 0$ and that $c(\cdot)$ is strictly increasing in its argument with $c(0) = 0$ and $c(\infty) = \infty$. At the time that consumer decides whether to become a shopper, she observes how many firms are in the market and how many total products are offered which we define as $L \equiv \sum_{j=1}^{n} \ell_j$. If the consumer chooses not to search, she randomly visits one of the n firms with equal probability and is paired with one of its products.

An important consideration in this model is that because product lines potentially contain multiple options, search may not only take place between stores, but also within them. For example, sequential search across stores could be associated with an all-or-nothing or sequential search process within each store. How fine (i.e., discriminating) the search process is depends on both dimensions and dictates optimal firm and consumer behavior. For analytical tractability, we start by assuming that search within stores is all-or-nothing in nature. That is, even for the sequential search case across stores, the consumer learns about all products at any store she visits. However, as we show later, our main findings are preserved without this assumption, and are in fact magnified by finer search within stores.

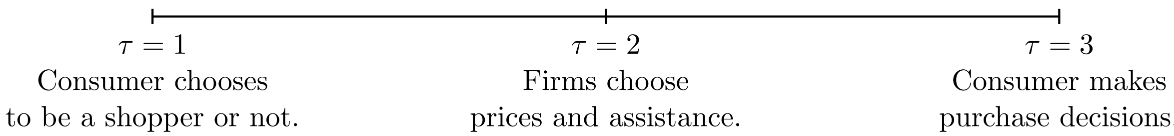

Figure 1: Timeline for search fatigue stage game. At the beginning of the game ($t = 0$), each firm chooses the length of its product line. Then, in each following period $t \in \{1, 2, \ldots\}$, the choices of the consumer and firms take place according to this timeline. First, at $\tau = 1$, the consumer decides whether she wants to become a shopper. At $\tau = 2$, the firms choose their prices whether to provide consumer assistance or not. Finally, at $\tau = 3$, the consumer chooses whether to proceed with her purchase or walk away.

Define J_t to be the set of stores that a consumer visits in period t. If a consumer visits a firm's store and the firm offers assistance ($a_j = 1$), she is directed to the special product, even if she is a non-shopper and was randomly allocated to another product offered by the firm. In this case, she saves energy and this experience does not impact future search costs. If a firm chooses $a_j = 0$ and a shopper sorts through that firm's products, the consumer suffers fatigue which may affect her decision-making in the future.

2.1.3 Timing

The timing of the game is as follows. At the beginning of the game ($t = 0$), each firm chooses ℓ_j. Then, each period $t \in \{1, 2, \ldots\}$ is sub-divided into three parts. At $\tau = 1$, the consumer decides whether to become a shopper. Then, at $\tau = 2$, the firms set prices for each of their products and choose whether to offer consumer assistance.[6] At $\tau = 3$, the consumer makes her purchase decision. The consumer makes a purchase only if her utility is weakly greater than walking away. The timing of these three stages of each period is depicted in Figure 1.

2.2 Results

We define a subgame perfect equilibrium by the tuple of vectors $(\ell^*, F_t^*(p), a_t^*, s_t^*)$. The vector $\ell^* \equiv [\ell_1^*, \ldots, \ell_n^*]$ lists the product line choices that the firms choose at $t = 0$. The vectors $F_t^*(p)$

[6] We make the assumption that firms set prices after the consumer chooses whether to search to capture the reality that firms often offer discounts after the consumer arrives at their store. In an alternative model in which firms post prices before the consumer searches, but then can offer discounts, a rational consumer will take into account each firm's ex post behavior when deciding whether to use her search technology.

and a_t^* are the time-varying equilibrium pricing and advice strategies that the firms use. As we will show shortly, in the single consumer case, pricing entails pure strategies, whereas with heterogeneous fatigue (Section 3), the firms utilize mixed strategies. Finally, s_t^* is the time-dependent decision by the consumer to become a shopper.

We begin by analyzing the monopoly case where $n = 1$. This will serve as a benchmark of comparison to the oligopoly case.

Proposition 1. *(Monopoly) Suppose that* $n = 1$*. Then, there exists an equilibrium in which* $\ell^* = 1$*,* $p^{1,*} = \bar{q}$ *for all* t*, no search takes place,* $s_t^* = 0$*, and no assistance is given,* $a_t^* = 0$*. Consumer surplus is zero. The firm's expected discounted profit (and total welfare) is* $\Pi^* = \frac{1}{1-\delta}\bar{q} - \kappa$*.*

When there is a single monopolist firm, there is no product proliferation. Since the monopolist collects all of the rents from production, there is no reason to offer suboptimal products. Second, there is no need for the consumer to become a shopper. Because there is only one firm and one valuable product, no effort is wasted on search and there is no fatigue.[7] Thus, the monopoly case is most efficient from a welfare standpoint. Since prices are merely transfers, zero costs are expended on extra product proliferation and sorting products.[8] This efficiency is not retained in the oligopoly setting.

Proposition 2. *(Oligopoly Search Fatigue) Define* $\bar{L}$ *to be the smallest integer such that* $\bar{q} \leq c(\bar{L})$*. For all-or-nothing search, in equilibrium there exists a unique* $L^* \equiv \sum_{j=1}^{n} \ell_j^* = \bar{L}$*. For sequential search,* $\ell_j^* = \bar{L}$ *for all* j *such that* $L^* = n\bar{L}$*. For either all-or-nothing or sequential search across stores the equilibrium has the following features:*

1. *In all odd-numbered periods* ($t = 1, t = 3, \dots$)*, the consumer becomes a shopper,* $s^* = 1$*,* $a_j^* = 0$ *for all* j*, and* $p_j^{1,*} = 0$ *for all* $j \in N$*. In all even-numbered periods* ($t = 2, t = 4, \dots$)*,*

[7] There also exists an equilibrium in which the consumer searches every other period because $c(0) = 0$. But, it is still the case that $\ell^* = 1$, $p^{1,*} = \bar{q}$ for all t, and no assistance is given, $a_t^* = 0$. Consumer and producer surplus are the same.

[8] Traditional welfare losses of monopoly that arise from the reduction of quantity sold to consumers are absent in our model. Here, the monopolist is able to extract all the surplus from the consumers and thus does not exclude any consumer who has a valuation above the marginal cost of production of the good. Also, in the monopoly case, there is no incentive to over-produce or differentiate, as there typically is in models in which the threat of competition induces excess product proliferation (e.g., Schmalensee, 1978).

no consumer search occurs, $s^* = 0$*,* $a_j = 1$ *for all* j*,* $p_j^{1,*} = \bar{q}$ *for all* j*, and* $p_j^{m,*} > 0$ *for all* $j \in N, m \in \ell_j$ *such that* $m > 1$*.*

2. *Each firm earns discounted expected profits equal to* $\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - \ell_j^* \kappa$ *and the consumer's expected discounted surplus is* $U^* = \frac{1}{1-\delta^2}\bar{q}$*. Total welfare is equal to* $\frac{1}{1-\delta}\bar{q} - L^*\kappa$*.*

According to Proposition 2, search, prices, and consumer assistance are time-varying. Even though the consumer might like to pace herself when faced with a marathon of lifetime choices, she finds this suboptimal.[9] This is because the firms have an incentive to produce enough products to ensure that she becomes sufficiently fatigued when searching and has to give up future surplus while she recovers. Therefore, the consumer receives $\frac{1}{1-\delta^2}\bar{q}$ of the total gains from trade and the firms earn $\frac{\delta \bar{q}}{(1-\delta^2)}$. Compared to the monopoly case above, the deadweight loss drop in total welfare is $(L^* - 1)\kappa$, which is due to product proliferation and competition. Note, however, that there are no direct deadweight costs of search. First, the consumer only searches after resting and therefore her search costs are $c(0) = 0$. Second, after searching in the previous period she has costs of $c(\bar{L})$ and chooses not become a shopper.

The equilibrium characterization is not sensitive to our assumption about the random choice of product whenever the consumer does not become a shopper. In Appendix B, we relax this in a model of all-or-nothing search and instead assume that a consumer who is not a shopper in period t is matched to the same product that she purchased in period $t-1$. In this case, firms compete as loss leaders, but the same types of cycles arise in which we observe product proliferation, intermittent search, and price fluctuations.

There a few other important things to glean from Proposition 2. First, when search is all-or-nothing, while $\bar{L}$ is unique, a unique equilibrium does not exist. To see this, suppose $n = 4$ and $\bar{L} = 9$. There are many permutations in which the firms can produce that will constitute an equilibrium. As long as $\bar{L}$ products are produced by all the firms in the market, there is no reason for any one firm to deviate. This implies then that competition and product proliferation are natural substitutes when all-or-nothing search takes place. Since it takes a fixed number of $\bar{L}$

[9] The consumer has no reason to deviate from the equilibrium in Proposition 2. If she did so, she might rest in two subsequent periods. However, she has no incentive to do so because she has a search cost of zero. Resting twice, she would forego a current payoff of $\bar{q}$ and would face the identical decision next period.

products to fatigue the consumer, as competition increases, the requirements for each firm to offer wasteful products decreases.

Second, the severity of the product proliferation depends on how discriminating the consumer is in her search. Sequential search leads to more severe product proliferation. Since there is Bertrand pricing when the consumer is a shopper, she only visits one firm. Given this, each firm must produce $\bar{L}$ products to induce sufficient fatigue so that the consumer rests intermittently. There are $n\bar{L}$ products in the market which increases socially wasteful product proliferation and leads to a drop in welfare relative to all-or-nothing search.

Corollary 1. *Suppose that the consumer engages in sequential search among stores and also within them. Then, in any equilibrium, $\ell_j^* > \bar{L}$ for all j.*

When the consumer is able to sort through products one-by-one (which is the finest type of search possible) she may get lucky and find the special product with little effort. In this case, she will not get fatigued and will be able to become a shopper again next period. Based on the luck of the draw, the consumer might search in several periods in a row. However, to combat this, every firm will increase its product proliferation to make this less likely. As a result, a *finding the needle in a haystack* game will arise when the consumer is most discriminating. This again lowers social welfare due to product proliferation. In addition, the cycles that we characterize in Proposition 2 will still arise, but they are stochastic and depend on how lucky the consumer's search draws are.

We complete this section by relaxing our assumption that the consumer incurs a fixed cost to engage in a sequential search process. Instead, let us suppose that the consumer pays a variable cost $c(x_{t-1})$ for each firm she visits. We continue to assume that $c(\cdot)$ is strictly increasing in its argument, $x_0 = 0$, $c(0) = 0$, $c(\infty) = \infty$, and that all of the other aspects of the model remain the same.

Proposition 3. *(Search Fatigue with Variable Costs) Suppose that the consumer incurs a variable cost $c(x_{t-1})$ for each firm she visits. The equilibrium has the following features:*

1. *$\ell_j^* = 2$ for all j so that $L^* = 2n$.*

2. *In all odd-numbered periods* ($t = 1, t = 3, \dots$), $a_j^* = 0$ *for all* j, *and* $p_j^{1,*} = 0$ *for all* $j \in N$. *In all even-numbered periods* ($t = 2, t = 4, \dots$), $a_j = 1$ *for all* j, $p_j^{1,*} = \bar{q}$ *for all* j, *and* $p_j^{m,*} > 0$ *for all* $j \in N, m \in \ell_j$ *such that* $m > 1$.

3. *Each firm earns discounted expected profits equal to* $\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - 2\kappa$ *and the consumer's expected discounted surplus is* $U^* = \frac{1}{1-\delta^2}\bar{q}$. *Total welfare is equal to* $\frac{1}{1-\delta}\bar{q} - 2n\kappa$.

According to Proposition 3, deterministic price and assistance cycles still arise when the consumer pays incremental costs of search. After resting, her cost is $c(0) = 0$. Therefore, she has the freedom to sort through all of the products in the market, Bertrand pricing arises, and she extracts the full surplus $\bar{q}$. However, after she searches, she faces a positive cost of search. In this case the Diamond Paradox arises (Diamond, 1971). Firms set monopoly prices and the firm who attracts the consumer enjoys the full surplus.

Note, however, that this setting features two important differences. First, there is little product proliferation. Since any positive incremental search cost will yield the Diamond Paradox, only one superfluous product is necessary to prevent search in the subsequent period. This means that the loss in total welfare resulting from product proliferation is significantly less pronounced. Second, the nature of the equilibrium is sensitive to our assumption that $c(0) = 0$. If $c(0)$ was in fact positive, albeit lower than if the consumer had searched in a prior period, no cycles would arise. This is because the results of the Diamond Paradox are not sensitive to the magnitude of the search cost as long as it is positive. Therefore, if $c(0)$ was positive, no search would arise and there would be no fatigue either.

3 Aggregate Time-Varying Fatigue

In this section, we extend our analysis to consider heterogeneous fatigue under all-or-nothing search. We derive an equilibrium with a time-varying distribution of prices, product proliferation, and consumer assistance. We then consider the impact of brand loyalty: some consumers may have switching costs or inertia when deciding from which firm to buy.

3.1 Heterogeneous Fatigue

Consider an all-or-nothing search environment in which μ_t is the fraction of shoppers in any period t and $1-\mu_t$ is the fraction of non-shoppers. Non-shoppers are randomly paired with a firm. In the initial period $t=1$, denote the fraction of people who are rested by r. By construction, these are the consumers who may (but need not) choose to become shoppers in $t=1$. The remaining proportion $1-r$ start as non-shoppers because they are too fatigued to search in the first period. The setup is otherwise unchanged from Section 2. As before, the firms initially choose their product lines ℓ_j at $t=0$ and then the timing within each subsequent period $t \in \{1,2,\ldots\}$ proceeds according to the same timeline as shown in Figure 1.

Proposition 4. *(Heterogeneous Fatigue) Suppose that n firms compete in a dynamic all-or-nothing search game in which μ_t of the consumers are shoppers at any time t. Then, there exists a $\bar{\delta}$ such that for $\delta \leq \bar{\delta}$, there exists an equilibrium $(\ell^*, F_t^*(p), a_t^*)$ where $L^* = \bar{L}$ and all firms set their prices for the special product according to a continuous, monotonically increasing, time-varying distribution function $F_t(p)$ over the support $[p_t^*, \bar{q}]$. The distribution function is computed as*

$$F_t(p) = 1 - \left[\frac{(\bar{q}-p)(1-\mu_t)}{np\mu_t}\right]^{\frac{1}{n-1}}, \tag{1}$$

with lower bound

$$p_t^* = \frac{\bar{q}}{\frac{n\mu_t}{1-\mu_t}+1}.i \tag{2}$$

In equilibrium, $\mu_t = r$ for all odd periods $t \in \{1,3,\ldots\}$ and $\mu_t = 1-r$ otherwise. The lower bound p_t^* is monotonically decreasing in μ_t and n. The function $F_t(p)$ is monotonically increasing in μ_t. As $n \to \infty$, $p_t^* \to 0$ and $F_t(p) \to 0$ for all p. Assistance is given by $a_t^* = 0$ ($a_t^* = 1$) for searchers (non-searchers). Finally, total welfare is equal to $\frac{1}{1-\delta}\bar{q} - \bar{L}\kappa$.

According to Proposition 4, the fraction r of consumers search during $t=1$ and get the best deal in the market. Their payoff is $\bar{q}-p_{min}$. The firms provide no assistance to searching customers and as a result they are fatigued after analyzing $\bar{L}$ products. Given their fatigue, they do not search at time $t=2$. In contrast, each consumer in the fraction $1-r$ does not search at $t=1$ and is randomly paired with a firm. They each receive assistance and all purchase the special product from the firm that they visit. Their payoff is also almost surely positive and depends on their firm's draw from $F^*(p)$.[10] Since they do not search, however, they become rested at time $t = 2$ and search during that period. Therefore, in each period the roles of the consumers switch. There will be time-varying price distributions that take values in one of two states, alternating each period depending on r and $1 - r$.

Not surprisingly, in each period, the firms play a mixed strategy equilibrium when pricing their special product. This result is consistent with previous all-or-nothing search models (e.g., Salop and Stiglitz, 1977; Varian, 1980) and price dispersion results from the impossibility of a pure strategy equilibrium. In this case, however, the distribution of prices for the special products varies over time due to the heterogeneous and intermittent fatigue of the consumers. Only if $r = \frac{1}{2}$ will $F_t(p)$ be constant over time. As r departs from $\frac{1}{2}$, there is increasing variation across periods. In the extreme, when r is one, the firms will set prices as they did in Section 2, that is alternating between Bertrand competition and monopoly prices. Therefore, Proposition 4 nests our result on all-or-nothing search in Proposition 2 as a special case when consumers are homogeneous. Our previous results on total welfare are also preserved in the context of heterogenous fatigue. Because prices are merely transfers between firms and consumers and consumers only search when rested, the sole welfare-reducing effect of search results from the firms' decision to engage in product proliferation.

According to our comparative statics results, with more firms (higher n), the lower bound of the distribution decreases, consistent with more competition. However, the magnitude of this pro-competitive effect depends on the proportion of shoppers μ_t. It is straightforward to show that the cross-derivative $\frac{\partial^2 p_t^*}{\partial \mu \partial n}$ is positive if $\mu_t > \frac{1}{n+1}$, and negative otherwise. This means that if the measure of searchers is high, the competitive effect is magnified. But if the fraction of fatigued consumers is high, the effect is muted. Notwithstanding, in the limit as $n \to \infty$, even though the lower bound converges to zero, the firms all choose $p_j = \bar{q}$ almost surely. This is because increasing competition makes it less likely to be the low-price firm in the market. This seemingly paradoxical effect that an increase in competition may induce an increase in prices was first described by Rosenthal (1980).

[10] For this reason, any non-negative price for the redundant products may be a part of an equilibrium. Searchers and non-searchers alike are almost surely better off with a special product.

Note that Proposition 4 requires that δ be sufficiently small so that no consumer has a unilateral incentive to deviate from the equilibrium. Specifically, as long as δ is small enough, a rested consumer will not find it profitable to rest for an additional period and join the other group of consumers who have no choice but to rest in the current period. To show this, define $E[p|\mu_t]$ as the expected price a consumer pays at time t when she makes a purchase from a random firm and $E[\min\{p\}|\mu_t]$ as the expected minimum price a searching consumer expects to pay given that the firms are using $F(p, \mu)$. If a consumer is rested at time t, she will rest for an additional period if and only if

$$\Delta(\delta, \mu_t) = \delta(E[p|1-\mu_t] - E[\min\{p\}|1-\mu_t]) - (E[p|\mu_t] - E[\min\{p\}|\mu_t]) > 0, \tag{3}$$

which we call the cohort deviation payoff. Otherwise, the consumer will search this period and remain with her cohort. The first term of (3) is the discounted benefit of resting this period, while the second is the lost benefit of not searching in the current period. Clearly, for sufficiently small δ, there is no incentive for any consumer to switch cohorts.

What can we say about consumer and firm behavior when $\delta > \bar{\delta}$? In general, it is straightforward to show that $\Delta(\delta, \mu_t)$ is anti-symmetric around $\mu_t = \frac{1}{2}$, i.e.,

$$\Delta(\delta = 1, \mu_t) = -\Delta(\delta = 1, 1 - \mu_t).$$

Hence, if it is profitable for consumers from the proportion μ_t to switch, then the remaining $1 - \mu_t$ consumers will have no incentive to switch. Furthermore, it is easy to show that

$$\Delta(\delta, \mu_t = 0) = \Delta(\delta, \mu_t = \frac{1}{2}) = \Delta(\delta, \mu_t = 1) = 0.$$

Recall that the proportion of consumers who begin the game rested is r. When $r = 0$ ($r = 1$), the firms uniformly charge the monopoly price in odd (even) periods and price at marginal cost in even (odd) periods. In both cases, $\Delta(\delta = 1, \mu_t) = 0$ since the expected price and the expected minimum price coincide. Similarly, when $r = \frac{1}{2}$, we have $\mu_t = \frac{1}{2}$ and the firms use the same $F(p, \mu_t)$ in every period t and thus no consumer has any incentive to switch.

Away from these limits and for δ sufficiently large, one of two steady state equilibria may potentially arise if $\mu_t \in (0, \frac{1}{2})$ or $\mu_t \in (\frac{1}{2}, 1)$. The first is characterized by convergence to $\mu_t = \frac{1}{2}$

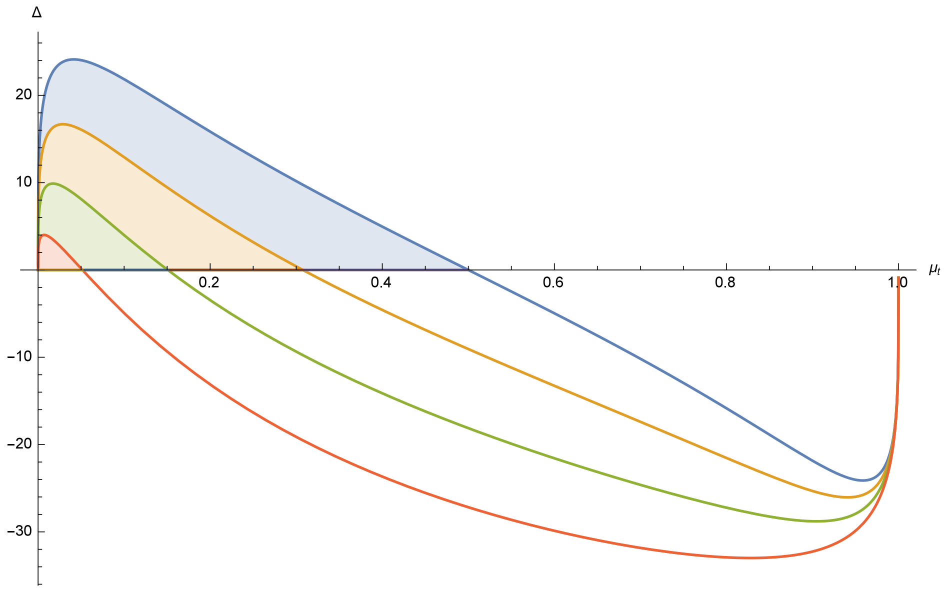

Figure 2: Cohort deviation payoff $\Delta(\delta, \mu_t)$ as a function of μ_t. The four curves are calculated for decreasing values of δ. From top to bottom, δ is equal to 1, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively. The other parameters are set at $\bar{q} = 100$ and $n = 5$.

for all t. In this case, consumers in the majority optimally switch into the other group, and consumers are split evenly in steady state. The second is characterized by all consumers becoming homogeneous, searching and resting together. Deriving analytical conditions under which one equilibrium rather than another arises remains elusive.[11] However, our numerical analysis reveals that the second equilibrium arises when δ is large enough.[12]

For example, consider Figure 2 where we plot the cohort deviation payoff $\Delta(\delta, \mu_t)$ as a function of μ_t for different values of δ. When $\delta = 1$ (top curve), for any $0 < \mu_t < \frac{1}{2}$ the cohort deviation payoff is positive. In this case, the members of the minority will find it optimal to join the other cohort. In steady state, consumers are homogeneous and equilibrium pricing mimics that of Proposition 2. However, as δ decreases to lower levels, there exists a region for $\mu_t < \frac{1}{2}$ for which the cohort deviation payoff is not sufficient to warrant switching cohorts. This is because the benefit of resting

[11]We are unable to prove in all generality that $\Delta(\delta = 1, \mu_t)$ is strictly positive and concave in μ_t for all $0 < \mu_t < \frac{1}{2}$ and hence $\Delta(\delta = 1, \mu_t) < 0$ and convex for all $\frac{1}{2} < \mu_t < 1$. This would establish that for $\delta > \bar{\delta}(\mu_t)$ all consumers searching with the minority of consumers would switch to searching with the majority of consumers. In that case, for $\mu_t < \frac{1}{2}$ all consumers will search in even periods and rest in uneven periods while the opposite would hold if $\mu_t > \frac{1}{2}$ and the price distribution used by the firms would vary according to Proposition 2.

[12]We have not uncovered an example in which the first type of steady state equilibrium arises, but it remains a theoretical possibility.

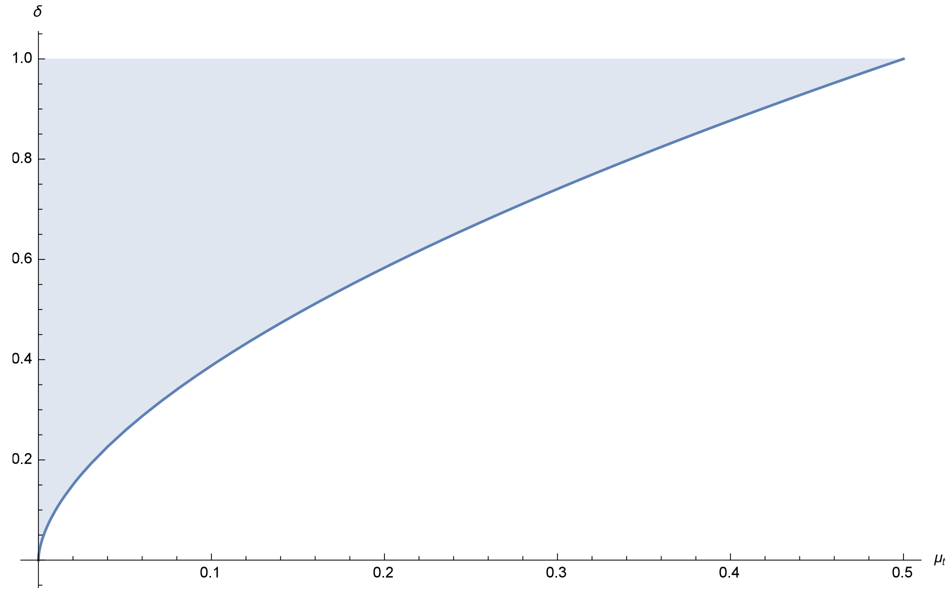

Figure 3: Critical threshold $\bar{\delta}(\mu_t)$ as a function of μ_t for $\bar{q} = 100$, $n = 5$. The shaded region contains μ_t-δ pairs for which consumers join together in a common cohort that searches and rests together. The unshaded region contains μ_t-δ pairs for which consumers remain in their separate cohorts.

today and searching tomorrow when the price distribution is more favorable is discounted at a higher rate. As δ decreases further, the deviation benefit decreases further and switching only occurs if there is a small minority (low μ_t).

Based on this, for any δ there will exist a critical μ above which no switching occurs and the steady state equilibrium is characterized by Proposition 4. To better see this, consider Figure 3 in which the critical threshold $\bar{\delta}(\mu_t)$ is plotted as a function of μ_t. This threshold is increasing in μ_t. The curve divides Figure 3 into two regions. For any given μ_t, if $\delta > \bar{\delta}$, a homogeneous steady state equilibrium arises with pricing as in Proposition 2 (shaded region). Otherwise, if $\delta < \bar{\delta}$, consumers do not deviate and the equilibrium takes the same form as in Proposition 4 (white unshaded region).[13]

[13] Note that because of the switch between fatigued and rested state the long-run (ergodic) search intensity of consumers will vary between periods regardless of whether it is homogeneous (Proposition 2) or heterogeneous (Proposition 4).

3.2 Brand Loyalty

So far in our analysis, we have made the somewhat unrealistic assumption that consumers (may) switch products every period and start afresh. Now, we relax that assumption and assume that consumers can develop brand loyalty, either due to inertia, switching costs that are outside the model, or relationships with someone at the firm.

Consider the same setup as Section 3.1 except that at any time t, a fraction $1-\lambda$ of consumers have brand loyalty. We assume that $1-\lambda$ is evenly distributed among the firms so that no firm has a brand advantage, and that λ is common knowledge among all firms and consumers. Of the remaining λ fraction of consumers, a proportion μ_t are rested and become shoppers during period t. The remaining $1-\mu_t$ fraction of λ are each randomly paired with a firm.

Proposition 5. *(Brand Loyalty) Suppose that n firms compete in a dynamic all-or-nothing search game in which $1-\lambda$ of the consumers have brand loyalty at any time t. Then, there exists a $\bar{\delta}$ such that for $\delta \leq \bar{\delta}$, there exists an equilibrium $(\ell^*, F_t^*(p), a_t^*)$ where $L^* = \bar{L}$ and all firms set their prices for the special product according to*

$$F_t(p) = 1 - \left[\frac{(\bar{q}-p)(1-\lambda\mu_t)}{np\lambda\mu_t}\right]^{\frac{1}{n-1}}, \tag{4}$$

over the support $[p_t^*, \bar{q}]$ with lower bound

$$p_t^* = \frac{\bar{q}}{\frac{n\lambda\mu_t}{1-\lambda\mu_t}+1}. \tag{5}$$

The lower bound p_t^* is monotonically decreasing in λ. The function $F_t(p)$ is monotonically decreasing in λ for all p. Assistance is given by $a_t^* = 0$ ($a_t^* = 1$) for searchers (non-searchers) and total welfare is equal to $\frac{1}{1-\delta}\bar{q} - \bar{L}\kappa$.

Much of the structure of the equilibrium with brand loyalty is unchanged from the results in Proposition 4. The fraction of searching consumers alternates in each period, the firms use a mixed strategy when choosing prices, and firms help non-searching consumers (including those with brand loyalty).

The distribution of prices is the only dimension that changes with brand loyalty. When brand loyalty is high (low λ), the lower bound of the distribution is higher, but the firms place less probability weight on high prices. As $\lambda \to 0$, $p_t^* \to \bar{q}$. In other words, when all consumers exhibit brand loyalty, every firm enjoys monopoly power and prices converge to $\bar{q}$ in both cohorts which effectively returns us to the result of Proposition 1. In contrast, when there is low brand loyalty, there is more competition for searchers and p_t^* is lower. However, given a lower chance of being the low-priced firm, all firms weight higher prices more within $F_t(p)$. This implies that there is likely to be more price dispersion in periods of low brand loyalty than when there is more brand loyalty.

4 Conclusion

In this paper, we analyzed how fatigue affects consumer search and firm behavior under monopoly and oligopoly. Our model variations focused on one industry and we found that search fatigue leads to product proliferation, time-varying prices, cyclical price dispersion, and consumer assistance.

Our analysis would apply equally well to a case in which a consumer searches for goods in multiple industries. In the context of our model, as long as there is a probability that the consumer searches for an item in each industry in each period, firms should still find it optimal to produce multiple products, and prices (or price distributions) should be time varying. Thus, we believe that our model's empirical predictions can be extended across industries as well, especially those in which consumers search for related items (e.g., loans, investments, and insurance).

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Appendix A

Proof of Proposition 1.

The proof proceeds by backward induction within each period and checking whether the firm or the consumer has a profitable deviation.

Suppose that $\ell = 1$. Then, the firm has no incentive to provide assistance. Therefore, $a = 0$ is consistent with an equilibrium. At $\tau = 2$, the firm chooses $p = \bar{q}$ since if $p > \bar{q}$ the consumer does not purchase and if $p < q$, the firm earns strictly less. At $\tau = 1$, the consumer does not search, since $\ell = 1$. Searching after periods of rest is not a profitable deviation even though $c(0) = 0$.

Now, we can check if choosing $\ell = 1$ at $t = 0$ is optimal for the firm. Since the firm's profit function is

$$\Pi^* = \frac{1}{1-\delta}\bar{q} - \kappa\ell,$$

choosing any $\ell > 1$ is not a profitable deviation. ■

Proof of Proposition 2.

The proof proceeds by backward induction within each period. There are L products offered in the market and n special products.

Suppose the consumer becomes a shopper in period t and selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^m = 0$ for all j and m. To show this, suppose that $p_j^m = 0$ for all j and m, and that firm j deviates by setting $p_j^m > 0$ for any m. For $m \geq 2$, the payoff to the consumer from purchasing such a product from firm j would be negative which would lead to no sale. For $m = 1$, if $p_j^1 > 0$, the consumer would purchase the special product from another firm. Therefore, increasing prices is not a profitable deviation. In this case, the consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$. All firms earn zero profits.

Suppose the consumer does not becomei a shopper in period t. The consumer is willing to buy the product to which she was allocated as long as it offers non-negative utility. If the consumer is randomly allocated to firm j and $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits. Each firm's expected per period profit is equal to $\frac{\bar{q}}{n}$. The consumer earns zero surplus.

Hence, the consumer becomes a shopper in period t if and only if $\bar{q} > c(x_{t-1})$. If the consumer did not become a shopper in period $t-1$ (i.e., $x_{t-1} = 0$), she becomes a shopper in period t because $\bar{q} > 0$. Let $\bar{L}$ be the smallest integer such that $\bar{q} \leq c(\bar{L})$. If $L < \bar{L}$, the consumer becomes a shopper in every period and each firm earns zero discounted expected profits. If $L \geq \bar{L}$, the consumer becomes a shopper in all odd-numbered periods ($t = 1, t = 3, \ldots$) and does not search in all even-numbered periods ($t = 2, t = 4, \dots$). In this case, each firm j earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - \ell_j \kappa, \tag{A1}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{A2}$$

Consider that search is all-or-nothing. We now show that $L^* = \bar{L}$ is unique in equilibrium and that $a_j = 0$ in odd-numbered periods. Suppose firms except j choose to produce a total of $x < \bar{L}$ products. It is straightforward to show that firm j prefers to produce $\bar{L} - x$ products to deter search in all even-numbered periods. Now, suppose that the total number of products offered in the market is equal to $\bar{L}$. If a particular firm j produces $\ell_j^* + 1$ instead of ℓ_j^* products, it incurs an extra cost κ and thus reduces its expected discounted profits. If it instead produces only the special product, it avoids paying $(\ell_j^* - 1)\kappa$ in product line costs, but its (expected) per-period profit drops to zero. Because κ is small, this is never a profitable deviation. Finally, when the consumer searches in odd-numbered periods, firm j has a strict incentive to set $a_j = 0$ since $c(\bar{L} - 1) < \bar{q}$.

Now consider that search is sequential. Since the consumer will only visit one firm when rested, each firm has a strict incentive to produce $\bar{L}$ products. If $\ell_j < \bar{L}$ and the consumer visits firm j, she will become a shopper in the next period and firm j will lose $\frac{\delta \bar{q}}{n}$. Because $\kappa(\bar{L}) < \frac{\delta \bar{q}}{n}$, the firm has no incentive to deviate from producing $\bar{L}$. Likewise, if firm j produces $\bar{L} + 1$ instead of $\bar{L}$ products, it incurs an extra cost κ and reduces its expected discounted profits. This is not a profitable deviation. ■

Proof of Corollary 1.

The logic of Proposition 2 for sequential search across firms holds here as well, except that each firm has an incentive to produce more than $\bar{L}$ products. To see this, suppose each firm produces exactly $\ell_j^* = \bar{L}$ and the consumer sorts products one-by-one. Assume that the consumer randomly chooses a product each time she decides to examine a subsequent one. Define $P(\ell)$ as the probability that the consumer chooses the special product after at least $\bar{L}$ draws. By construction, $P(\ell)$ is increasing in ℓ for $\ell \geq \bar{L}$. With probability $1 - P(\ell)$, the consumer becomes a shopper again at $t + 1$ and each firm loses an expected surplus of $\frac{\delta \bar{q}}{n}$. Because $\kappa < [P(\bar{L} + 1) - P(\bar{L})]\frac{\delta \bar{q}}{n}$, each firm has a strict incentive to produce more than $\bar{L}$ products. ■

Proof of Proposition 3.

Suppose the consumer searches sequentially with variable cost $c(0) = 0$ and selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^m = 0$ for all j and m. The logic follows the same way as in the proof of Proposition 2 above. In this case, the consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$. All firms earn zero profits.

Suppose the consumer has a variable search cost of $c(x_{t-1}) > 0$. Then, following Diamond (1971), it is an equilibrium for each firm j with ℓ_j to set prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$. In such case, each firm has an incentive to provide advice $a_j = 1$ if the consumer is randomly paired with one of its non-special products. Taking this into account, there is no profitable deviation for the consumer to pay a positive search cost. In this case, the firm that attracts the consumer in this period earns $\bar{q}$ and all other firms earn zero profits. Each firm's expected per period profit is equal to $\frac{\bar{q}}{n}$. The consumer earns zero surplus.

Hence, the consumer searches when $c(0) = 0$ and does not search otherwise. Therefore, it is optimal for each firm j to produce two products. If firm j deviates and produces only the special product, and the consumer visits this store, firm j loses an expected surplus of $\frac{\delta\bar{q}}{n}$ next period. Since $\kappa < \frac{\delta\bar{q}}{n}$, the firm has no incentive to deviate from producing two products. Likewise, if firm j produces three products instead, it incurs an extra cost κ and reduces its expected discounted profits. This is not a profitable deviation.

Given this, the consumer searches in all odd-numbered periods ($t = 1, t = 3, \ldots$) and does not search in all even-numbered periods ($t = 2, t = 4, \ldots$). Each firm j earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta\bar{q}}{n(1-\delta^2)} - \ell_j\kappa, \tag{A3}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{A4}$$

■

Proof of Proposition 4.

Outline of proof: The proof proceeds by backward induction within each period. In each period, we first consider consumer buying behavior and the consumer assistance offered by each firm conditional on identifying which consumers are searching. Following that, we consider the firms' pricing strategies. Working backward, we then consider the search decision by consumers. Finally, we show the existence of an equilibrium $(\ell^*, F_t^*(p), a_t^*)$.

Step One: Buying behavior and consumer assistance

At any time t, μ_t-type consumers identify all products and prices in the market and choose the one that gives the highest payoff. By construction, $x_t = L$, so that their next period search cost is

$c(L)$. When a firm identifies a searching consumer, it chooses $a_j = 0$ as there is a cost to lowering future search costs and no benefit to giving assistance. At time t, $(1 - \mu_t)$-type consumers are randomly paired with a firm. In this case, the firm will offer $a_j = 1$ and direct the consumer to the product that is most profitable for the firm. As we will show shortly, in equilibrium this is the special product.

Step Two: Pricing

First, let us consider the price of the special product and assume that the firm always directs $(1 - \mu_t)$-types to this product. Eventually, we will show that this is indeed always optimal in equilibrium. Define J^* as the set of firms who quote the lowest price for the special product and n_{j^*} as the number of firms in J^*. Then, the payoff function for each firm $j \in N$ is

$$\max_{p_j \in [0,\bar{q}]} \quad \pi_j(p_j) = p_j Q_j, \tag{A5}$$

where the expected demand Q_j is calculated as

$$Q_j = \frac{\mu_t \;\; 1_{\{j \in J^*\}}}{n_{j^*}} + \frac{1 - \mu_t}{n}.$$

Given this, the payoff to each firm is continuous, except when its price is the lowest and equal to at least one of its competitors.

We prove existence of a symmetric mixed-strategy equilibrium by appealing to Theorem 5 in Dasgupta and Maskin (1986). Using their notation, let $A_j = [0, \bar{q}]$ be the action space for firm j and let $a_j \in A_j$ be a price in that space. As such, A_j is non-empty, compact, and convex for all j. Define $A = \times_{j \in N} A_j$ and $a = (a_1, \ldots, a_n)$. Let $U_j : A \to \mathbb{R}$ be defined as the profit function in (A5). Define the set $A^*(j)$ by

$$A^*(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \; s.t. \; p_j = p_i\}$$

and the set $A^{**}(j) \subseteq A^*(j)$ by

$$A^{**}(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \; s.t. \; p_j = p_i = p_{min} > 0\}.$$

Therefore, the payoff function U_j is bounded and continuous, except over points $\bar{a} \in A^{**}(j)$. The sum $\sum_{j \in N} U_j(a)$ is continuous since discontinuous shifts in demand from informed consumers between firms at points in $A^{**} = \times_{j \in N} A^{**}(j)$ occur as transfers between firms who have the same low price in the industry. Finally, it is straightforward to show that $U_j(a_j, a_{-j})$ is weakly lower semi-continuous. Since any time $p_i = p_j = p_{min}$, firm i and j share the demand, there exists a $\lambda \in [0, 1]$ large enough such that

$$\lambda[(p_j - \epsilon)\mu_t + \frac{(p_j - \epsilon)(1 - \mu_t)}{n}] + (1 - \lambda)\frac{(p_j + \epsilon)(1 - \mu_t)}{n} \geq \frac{p_j \mu_t}{2} + \frac{p_j(1 - \mu_t)}{n}, \tag{A6}$$

for ϵ arbitrarily small. Rearranging and letting $\epsilon \to 0$ yields

$$\lambda p_j \mu_t \geq \frac{p_j \mu_t}{2}, \tag{A7}$$

which is true for all $\lambda \geq \frac{1}{2}$. Therefore by Theorem 5 in Dasgupta and Maskin (1986), there exists a symmetric mixed-strategy equilibrium for this subgame, conditional on the firms always directing non-searching consumers to the special product.

Now, we can prove properties about $F^*(p)$, again conditional on the firm always directing consumers to the special product.

i. Continuity: Suppose that there did exist a countable number of mass points in the distribution of $F^*(p)$. Then, for any $p > 0$, we can find a mass point p' and an $\epsilon > 0$ such that $f^*(p') = a > 0$ and $f^*(p' - \epsilon) = 0$. Now consider a deviation by firm j to choose $\hat{F}(p)$ such that $\hat{f}(p') = 0$ and $\hat{f}(p' - \epsilon) = a$. Since $E[\pi_j(p)]$ using $F^*(p)$ is strictly less than using $\hat{F}(p)$, this would be a profitable deviation. Last, there can never be a mass point at $p = 0$ since it is a dominated strategy to choose $p = 0$. Therefore, in equilibrium, no mass points can exist.

ii. Strict monotonicity (Increasing): Suppose there exists an interval $[p_a, p_b]$ within $[0, \bar{q}]$ such that $F(p_b) - F(p_a) = 0$ and that $F(p_a) > 0$. Then, for any $\hat{p}$ such that $p_a < \hat{p} < p_b$, $[1 - F(\hat{p})]^{n-1} = [1 - F(p_a)]^{n-1}$. Since $\hat{p}[1 - F(\hat{p})]^{n-1} > p_a[1 - F(p_a)]^{n-1}$ and $\hat{p}[1 - (1 - F(\hat{p}))^{n-1}] > p_a[1 - (1 - F(p_a))^{n-1}]$, then there exists a profitable deviation. Thus, $F(p_b) - F(p_a) \neq 0$ for any interval $[p_a, p_b]$ within $[0, \bar{q}]$.

Given continuity and strict monotonicity, we can write the symmetric $F(p)$ explicitly. For any price p that a firm may choose,

$$\pi_j(p) = p\mu_t[1 - F(p)]^{n-1} + \frac{p(1 - \mu_t)}{n}. \tag{A8}$$

Since each firm needs to be indifferent between setting an price over a support $[p^*, \bar{q}]$, we can write

$$p\mu_t[1 - F(p)]^{n-1} + \frac{p(1 - \mu_t)}{n} = \frac{\bar{q}(1 - \mu_t)}{n}. \tag{A9}$$

Rearranging yields the expression in (1). We can then solve

$$p^*\mu_t + \frac{p^*(1 - \mu_t)}{n} = \frac{\bar{q}(1 - \mu_t)}{n} \tag{A10}$$

for p^* which yields (2). Finally, inspecting (2), it is clear that $p^* > 0$ for any $\mu_t < 1$. Therefore, the firms will always direct non-searching consumers to the special product since they don't make positive profits by selling alternative products.

The comparative statics in Proposition 4 regarding μ_t and n are derived by straightforward differentiation. Taking the limit of $1 - F_t(p)$ yields

$$\lim_{n\to\infty}\left[\frac{(\bar{q}-p)(1-\mu_t)}{np\mu_t}\right]^{\frac{1}{n-1}} = \lim_{n\to\infty}\left[\frac{(\bar{q}-p)(1-\mu_t)}{p\mu_t}\right]^{\frac{1}{n-1}} \lim_{n\to\infty}\left[\frac{1}{n}\right]^{\frac{1}{n-1}} \to 1$$

which implies that as $n \to \infty$, $F_t(p) \to 0$ for all p.

Step Three: Consumer Search Decision

For the consumers with $x_{t-1} > 0$, they will search if and only if

$$\bar{q} - E[p_{min}|F(p)] > c(x_{t-1}). \tag{A11}$$

For now, let us suppose that the firms choose their product lines so that (A11) does not hold. Indeed, we will show this to be the case in step four below.

Given this, any consumer with $x_{t-1} = 0$ has the option to search since $c(0) = 0$ or rest again and search in the next period with those currently with $x_{t-1} > 0$. Let us consider this choice when the proportion of currently rested consumers is $\mu_t = r$. When the consumer decides to follow the equilibrium strategy to search this period, his expected discounted payoff is

$$q - E[\min\{p\}|r] + \delta(q - E[p|1-r]) + \delta^2(q - E[\min\{p\}|r]) + \delta^3(q - E[p|1-r]) + \dots \tag{A12}$$

where $E[\min\{p\}|r]$ denotes the expected minimum price when a proportion r of consumer searches. If he deviates and rests again, his payoff is

$$q - E[p|r] + \delta(q - E[\min\{p\}|1-r]) + \delta^2(q - E[p|r]) + \delta^3(q - E[\min\{p\}|1-r]) + \dots \tag{A13}$$

Note that the cost of search does not enter this payoffs since $c(0) = 0$. Thus, from equations (A12) and (A13) the consumer has an incentive to deviate if and only if

$$\Delta(\delta, r) = -(E[p|r] - E[\min\{p\}|r]) + \delta(E[p|1-r] - E[\min\{p\}|1-r]) > 0. \tag{A14}$$

By definition, we have $E[p|r] > E[\min\{p\}|r]$ and $E[p|1-r] - E[\min\{p\}|1-r]$. Thus, for sufficiently small δ this inequality is not satisfied and the consumer does not find it profitable to deviate from the equilibrium strategy. Therefore, there exists a $\bar{\delta}$ such that if $\delta < \bar{\delta}$, $\mu_t = r$ for all odd periods $t \in \{1, 3, \dots\}$ and is equal to $1 - r$ otherwise.

Step Four: Firms Choice of Product Lines

Given that $c(\cdot)$ is strictly increasing in its argument, there exists an $\bar{L}$ such that $\bar{q} - E[p_{min}|F(p)] < c(\bar{L})$, so the consumer does not search. With the condition that κ is small, proving that any ℓ^* that induces $L = \bar{L}$ follows the same logic as in Proposition 2. ■

Proof of Proposition 5.

The proof follows the exact same logic as the proof of Proposition 4. The only difference is the computation of $F_t(p)$.

In any period t, for any price p that a firm may choose,

$$\pi_j(p) = p\mu_t\lambda[1 - F_t(p)]^{n-1} + \frac{p\lambda(1-\mu_t)}{n} + \frac{p(1-\lambda)}{n}. \tag{A15}$$

Since each firm needs to be indifferent between setting an price over a support $[p^*, \bar{q}]$, we can write

$$p\mu_t\lambda[1 - F_t(p)]^{n-1} + \frac{p\lambda(1-\mu_t)}{n} + \frac{p(1-\lambda)}{n} = \frac{\bar{q}(1-\mu_t)}{n} + \frac{\bar{q}(1-\lambda)}{n}. \tag{A16}$$

Rearranging yields the expression in (4). We can then solve

$$p^*\mu_t + \frac{p^*(1-\mu_t)}{n} + \frac{p^*(1-\lambda)}{n} = \frac{\bar{q}(1-\mu_t)}{n} + \frac{\bar{q}(1-\lambda)}{n} \tag{A17}$$

for p^* which yields (5).

The comparative statics regarding λ in Proposition 5 are derived by straightforward differentiation. ■

Appendix B

B.1 Inertia

Let us reconsider the all-or-nothing search model posed in Section 2 with the following differences. To make the analysis easier, suppose that in each period, every firm chooses prices from $[0, \infty)$ for each of its products. More interestingly, let us relax the assumption regarding random allocation of the consumer to products in the market when she does not search. Let us suppose that for any $t \geq 2$, if the consumer does not search, she does not incur a cost and remains with the firm with whom she previously transacted. We call this the consumer's *incumbent* firm.

Proposition B1. *(Inertia) Suppose that n firms compete in a dynamic all-or-nothing search setting with inertia. Then, there exists an equilibrium with $L^* = \bar{L}$ and*

(i) In all odd-numbered periods ($t = 1, t = 3, \dots$), the consumer searches, $a_j^* = 0$ for all j, and $p_j^{m,*} = 0$ for all $j \in N, m \in \ell_j$.

(ii) In all even-numbered periods ($t = 2, t = 4, \dots$), no consumer search occurs, $a_j = 1$ for all j, $p_j^{1,*} = \bar{q}$ for all j, and $p_j^{m,*} > 0$ for all $j \in N, m \in \ell_j$ such that $m > 1$.

Each firm earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1 - \delta^2)} - \ell_j^* \kappa, \tag{B1}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1 - \delta^2}. \tag{B2}$$

Proof of Proposition B1.

Suppose that there are $L > n$ products offered in the market and that the consumer does not search in period $t = 1$. The consumer is willing to buy the product to which she was allocated as long as it offers non-negative utility. If the consumer is randomly allocated to firm j and $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits during that period and the consumer earns zero surplus.

Suppose the consumer does not search in period $t \geq 2$ and remains with her incumbent firm. Again, the consumer is willing to buy a product as long as it offers non-negative utility. If $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits in that period and the consumer earns zero surplus.

Now, consider that the consumer searches in period t and selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^1 = 0$ for all j. To see this, assume that $p_j^1 = 0$ for all j, and that firm k deviates by setting $p_k^1 > 0$. Since $p_k^1 > 0$, the consumer would purchase the special product from another firm. Therefore, increasing prices is not a profitable deviation.

Hence, the consumer searches in period t, if and only if $\bar{q} > c(x_{t-1})$. If the consumer did not search in period $t-1$ (i.e., $x_{t-1} = 0$), she searches in period t because $c(0) < \bar{q}$. Recall that $\bar{L}$ is the smallest integer such that $\bar{q} \leq c(\bar{L})$. If $L < \bar{L}$, the consumer searches in every period and each firm earns zero discounted expected profits. If $L \geq \bar{L}$, the consumer searches in all odd-numbered periods ($t = 1, t = 3, \dots$) and does not search in all even-numbered periods ($t = 2, t = 4, \dots$). In this case, each firm j earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - (\ell_j - 1)\kappa, \tag{B3}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{B4}$$

Suppose that in equilibrium all other $n-1$ firms choose to produce a total of $x < \bar{L}$ products. We now show that firm j prefers to produce $\bar{L} - x$ products to deter search in all even-numbered periods. The assumption that κ is small assures this to be the case. Further, when the consumer searches in odd-numbered periods, firm j has a strict incentive to set $a_j = 0$ since $c(\bar{L} - 1) < \bar{q}$.

Suppose that the total number of products offered in the market is equal to $\bar{L}$. If a particular firm j produces $\ell_j^* + 1$ instead of ℓ_j^* products, it incurs an extra cost κ and thus reduces its expected discounted profits. Now, suppose that the firm only produces the special product. It avoids paying $(\ell_j^* - 1)\kappa$ in product line costs, but its (expected) per-period profit drops to zero. Because κ is small, this is never a profitable deviation.

Hence, there is a unique equilibrium number of products $L = \bar{L}$.